

# Detecting service provider alliances on the choreography enactment pricing game\*

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## Abstract

We present the choreography enactment pricing game, a cooperative game-theoretic model for the study of scheduling of jobs using competitor service providers. A choreography (a peer-to-peer service composition model) needs a set of services to fulfill its jobs requirements. Users must choose, for each requirement, which service providers will be used to enact the choreography at lowest cost. Due to the lack of centralization, vendors can form alliances to control the market. We show a novel algorithm capable of detecting alliances among service providers, based on our study of the bargaining set of this game.

## 1 Introduction

Modern distributed systems are usually modeled and described by a service-oriented architecture that is completely distributed. The ongoing transition from the current service *orchestration* model for service composition — where the ensemble of services are composed as an executable business process, controlled by a single organization (the orchestrator) — to the service *choreography* composition model [21] — that describes a non-executable protocol for peer-to-peer interactions among different organizations.

This new model is not only more robust and scalable, but also more collaborative. The popularization of service choreography model enforces interoperability and loose coupling by reflecting obligations and constraints among different parties. This trend can be seen as a disruptive change on distributed software development, creating new opportunities for resource sharing among different organizations.

One of the implications of this trend is that even if a user chooses one service vendor (e.g., a cloud computing provider) to execute its application, there is nothing that prevents the vendor to subcontract resources from other vendors and scatter the application services among those vendors, creating a collaborative platform composed of resources from different organizations.

Beyond the complexities of the management of composite services (design, provisioning, etc.), we consider the problem from an economic point of view. Since service vendors are not regulated, both cooperative and non-cooperative behavior may be expected. This can potentially lead to the formation of alliances — i.e., the formation of groups of similar independent organizations, who join together to control prices and/or limit competition.

In this work we use tools and techniques from cooperative game theory [10] in order to analyze the stability and the alliance formation on this new scenario; a problem we call the *choreography enactment pricing game*.

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## 2 Problem statement

In this paper, we study how task assignment between independent service vendors can lead to the establishment of alliances on unregulated economies. We call *organization* a vendor, its physical computational resources and the users subscribed to it.

To state the problem, we will use the standard notation for scheduling on multi-organization platforms [8]. More formally, we have a set of  $N$  different organizations. Each organization  $O^{(k)}$ ,  $1 \leq k \leq N$ , has  $m^{(k)}$  machines that can be used to execute jobs submitted by its users or by other organizations.

A user must choose an organization to execute its jobs. The organization will work as a service broker, choosing which jobs it will execute on its own resources and which jobs will have its execution delegated to other organizations. The set of all jobs submitted from users of organization  $O^{(k)}$  is denoted by  $\mathcal{J}^{(k)}$ .

An organization also have costs associated to the execution of the jobs assigned to its machines. It is, however, free to schedule these jobs in any way they want, usually to optimize some given objective (common to all organizations), such as execution cost or global performance. We denote by  $\text{cost}_{\text{local}}^{(k)}$  the cost of the execution of jobs from  $O^{(k)}$ 's users when applying the optimal scheduling according to the organizations' own objective.

Organizations have the freedom to collaborate, organizing themselves to share their resources, jobs and to redistribute costs. An organization that choose to not cooperate must execute all jobs from its users on its own resources, paying  $\text{cost}_{\text{local}}^{(k)}$  to do so. A set  $\mathcal{C}$  of collaborating organizations share all their machines in order to execute the jobs in set  $\mathcal{J}^{(\mathcal{C})} = \cup_{k \in \mathcal{C}} \mathcal{J}^{(k)}$ . They are free to devise a scheduling that optimize their own objective using their combined resources. We denote as  $\text{cost}^{(\mathcal{C})}$  the cost of the optimal schedule. For the set  $\mathcal{J}^{(\mathcal{C})}$ , the total price that all organizations are ready to pay is given by  $p(\mathcal{C}) = \sum_{k \in \mathcal{C}} \text{cost}_{\text{local}}^{(k)}$ .

The payoff of a set of organizations  $\mathcal{C}$  is given as a function of the price charged by each organization to their users  $p(\mathcal{C}) = \sum_{k \in \mathcal{C}} \text{cost}_{\text{local}}^{(k)}$ . The difference between the sum of the costs that each organization would have if they were all alone, and the optimal cost achieved when these organizations are collaborating is known as *utility*  $v(\mathcal{C})$  and is given by:

$$v(\mathcal{C}) = p(\mathcal{C}) - \text{cost}^{(\mathcal{C})} \quad (1)$$

Using the terminology of cooperative game theory,  $v$  is the *characteristic function* of the game. Note that if  $\mathcal{C}$  is only composed of one organization  $k$ , then the utility is equal to zero ( $v(\mathcal{C}) = 0$ ).

We assume that all organizations have the same objective function. Two classical objective functions found on the literature are:

$(\sum_J C_J)$  [9]: Organizations are locally interested in minimizing the average completion time of their jobs. If scheduling its jobs in its own resources, the cost to schedule  $\mathcal{J}^{(k)}$  in  $O^{(k)}$ 's own resources is given by  $\text{cost}_{\text{local}}^{(k)} = \sum_{J \in \mathcal{J}^{(k)}} (C_J^{(k)})$ .

$(\sum_J E_J)$  [6]: Each organization  $O^{(k)}$ ,  $1 \leq k \leq N$ , can share several machines that supports continuous *dynamic speed scaling* (i.e., processors can operate at any arbitrary speed  $s$  that can be changed by the scheduler over time). A job  $J \in \mathcal{J}^{(k)}$ , is defined by its release date  $r_J^{(k)} = 0$ , its deadline  $d_J^{(k)}$  and its processing volume 1. The job with the biggest deadline of  $O^{(k)}$  is defined as  $d_{\max}^{(k)} = \max_i d_i^{(k)}$ . Job preemption is allowed. The energy consumption is given by the integral over time of the power function  $P(s(t)) = s(t)^\alpha$ , where  $s(t)$  is the speed in which the processor is running on time  $t$  and  $\alpha > 1$  is a constant real number that depends on the technical characteristics of the processor — usually  $\alpha \in [2, 3]$ . In other words, an organization  $O^{(k)}$  can execute its jobs consuming a total energy of  $E_{\text{local}}^{(k)}$  only using its own machines.

**Example 1.** Consider an instance of  $(\sum_J E_J)$  problem. Four organizations want to minimize its energy consumption. All these organizations have only one machine. Organizations  $O^{(1)}$  and  $O^{(2)}$  have respectively 19 and 7 jobs. Organizations  $O^{(3)}$  and  $O^{(4)}$  have 1 job, and all jobs have the same deadline date equals to 1. So, without cooperation, the local cost of organizations is the following :

$$\text{cost}_{\text{local}}^{(1)} = 19^\alpha, \text{cost}_{\text{local}}^{(2)} = 7^\alpha, \text{ and } \text{cost}_{\text{local}}^{(3)} = \text{cost}_{\text{local}}^{(4)} = 1^\alpha.$$

So each organization  $O^{(k)}$  has already to pay  $\text{cost}^{(\{k\})} (= p(\{k\}))$  to execute all these jobs. Assume that all four organizations form an alliance. This means that all jobs can be executed in all machines. The optimal schedule is as follow : 7 jobs are executed on each machine. So, the cost ( $\text{cost}^{(\{1,2,3,4\})}$ ) of the alliance  $\{1,2,3,4\}$  corresponds to the cost of the schedule of all jobs among all machines:  $\text{cost}^{(\{1,2,3,4\})} = (4 \cdot 7^\alpha)$ . So the energy consumption savings  $v(\{1,2,3,4\})$  is equal to  $(19^\alpha + 7^\alpha + 2) - 4 \cdot 7^\alpha$ . However, organizations  $O^{(3)}$  and  $O^{(4)}$  increase their own cost, and now their cost is equal to  $7^\alpha$ . So, these organizations should receive some payment in order their organization to have incentive to take part in the alliance. Now, we will give some energy consumption savings of different alliances:

$$\begin{aligned} v(\{1,2,3,4\}) &= (19^\alpha + 7^\alpha + 2) - 4 \cdot 7^\alpha; & v(\{2,3,4\}) &= (7^\alpha + 2) - 3 \cdot 3^\alpha \\ v(\{1,3,4\}) &= (19^\alpha + 2) - 3 \cdot 7^\alpha; & v(\{3,4\}) &= 0 \\ v(\{2,4\}) &= (7^\alpha + 1^\alpha) - 2 \cdot 4^\alpha; & v(\{2,3\}) &= (7^\alpha + 1^\alpha) - 2 \cdot 4^\alpha; \end{aligned}$$

Note that alliances  $\{1,2,3,4\}$  and  $\{1,3,4\}$  save the same amount of energy.

Let  $C_{\mathcal{SH}}^{(k)}$  be the global cost of the cooperative schedule  $\mathcal{SH}$  for organization  $O^{(k)}$ . The cooperative problem can then be stated as follows:

$$\text{Find } (x_1, \dots, x_N) \text{ such that, for all } k \ (1 \leq k \leq N), \ C_{\mathcal{SH}}^{(k)} - x_i \leq \text{cost}_{\text{local}}^{(k)} \\ \text{if such vector exists.}$$

The vector  $x$  represents the payment for each organization to have incentive to collaborate.

In this paper, we will focus on games in which binding agreements are possible. Now, we will focus on the payoffs of each organization, on the way which distributes the value of each coalition among its members.

### 3 Cooperative game theory [19]

A *cooperative game with transferable utility* is a pair  $([N], v)$  where  $[N] = \{1, \dots, N\}$  is a finite set of players and a *characteristic function*  $v : 2^{[N]} \rightarrow \mathbb{R}$  which associates each subset  $\mathcal{C} \subseteq [N]$  to a real number  $v(\mathcal{C})$  (such that  $v(\emptyset) = 0$ ). Each subset  $\mathcal{C}$  of  $[N]$  is called a *coalition*. The function  $v$  is a *characteristic function* of the game  $(V, v)$  and the *value* of coalition  $\mathcal{C}$  denoted by  $v(\mathcal{C})$  is the value that  $\mathcal{C}$  could obtain if they choose to cooperate. In these games, the value of a coalition can be redistributed among its members in any possible way.

#### 3.1 Revenue sharing mechanism

The challenge of a revenue sharing mechanisms is to find how to split the payoff  $v(\mathcal{C})$  among the players in  $\mathcal{C}$  while ensuring the stability of the coalition. A vector  $x = (x_1, \dots, x_{|V|})$  is said to be a *payoff vector* for a  $k$ -coalition  $\mathcal{C}_1, \dots, \mathcal{C}_k$  if  $x_i \geq 0$  for any  $i \in V$  and  $\sum_{i \in \mathcal{C}_j} x_i \leq v(\mathcal{C}_j)$  for any  $j \in \{1, \dots, k\}$ . We will focus on some particular payoff vectors, namely *imputations*.

**Definition 2.** A payoff vector  $x$  for a  $k$ -coalition  $\mathcal{C}_1, \dots, \mathcal{C}_k$  is said to be an *imputation* if it is efficient, — i.e.,  $\sum_{i \in \mathcal{C}_j} x_i \leq v(\mathcal{C}_j)$  for any  $j \in \{1, \dots, k\}$  — and if it satisfies the *individuality rationality property* — i.e.,  $x_i \geq v(\{j\})$  for any player  $j \in [N]$ .

The objective is to find a *fair* distribution of the value of the coalition (the payoff of each player corresponds to his actual contribution to the coalition) and also to ensure a *stable* coalition in such a way that no player or subset of players have incentive to leave the coalition.

**The objection** Let  $([N], v)$  be a cooperative game and  $x$  a payoff vector of this game. A pair  $(\mathcal{P}, y)$  is said to be *objection of  $i$  against  $j$*  if:

- $\mathcal{P}$  is a subset of  $[N]$  such that  $i \in \mathcal{P}$  and  $j \notin \mathcal{P}$  and
- if  $y$  is a vector in  $\mathbb{R}^{[N]}$  such that  $y(\mathcal{P}) \leq v(\mathcal{P})$ , for each  $k \in \mathcal{P}$ ,  $y_k \geq x_k$  and  $y_i > x_i$  (agent  $i$  strictly benefits from  $y$ , and the other members of  $\mathcal{P}$  do not do worse in  $y$  than in  $x$ ).

**The bargaining set.** [3, 17]

A pair  $(\mathcal{Q}, z)$  is said to be a *counter-objection* to an objection  $(\mathcal{P}, y)$  if:

- $\mathcal{Q}$  is a subset of  $[N]$  such that  $j \in \mathcal{Q}$  and  $i \notin \mathcal{Q}$  and
- if  $z$  is a vector in  $\mathbb{R}^{[N]}$  such that  $z(\mathcal{P}) \leq v(\mathcal{P})$ , for each  $k \in \mathcal{Q} \setminus \mathcal{P}$ ,  $z_k \geq x_k$  and, for each  $k \in \mathcal{Q} \cap \mathcal{P}$ ,  $z_k \geq y_k$  (the members of  $\mathcal{Q}$  which are also members of  $\mathcal{P}$  get at least the value promised in the objection).

Let  $([N], v)$  be a game with a coalition structure. A vector  $x \in \mathbb{R}^{[N]}$  is *stable* iff for each objection at  $x$  there is a counter-objection.

**Definition 3.** The bargaining set  $\mathcal{B}([N], v)$  of a cooperative game  $([N], v)$  is the set of stable payoff vectors that are individually rational, that is,  $x_i \geq v(\{i\})$ .

Note that it is sufficient to use the notion of imputations since the payoffs are individually rational.

The bargaining set concept requires objections to be immune to counter-objections, otherwise they are not considered as credible threats.

Now, we will go back to our problem and try to define appropriate characteristic function that reflects the outcome expected from coalition formation as described earlier in the problem statement section (Section 2).

### 3.2 The choreography enactment pricing game

The choreography enactment game models the cooperative game played by organizations. Their main objective is to form coalitions in order to schedule all jobs belonging to them having the lowest cost.

We consider particular objective functions for a schedule satisfying the following condition: the cost ( $\text{cost}^{([N])}$ ) is the sum of all costs ( $C_{\mathcal{SH}}^{(k)}$ ) of all organizations  $O^{(k)}$ . Note that this applies to the objective functions  $(\sum_J E_J)$  and  $(\sum_J C_J)$ .

Assume that  $[N]$  should form a coalition. Each organization  $O^{(k)}$  executes jobs having  $C_{\mathcal{SH}}^{(k)}$  as a cost. Its cost can be higher than its local cost  $\text{cost}_{\text{local}}^{(k)}$ .

In order to incentive organization  $O^{(k)}$  to join a coalition, the cost savings should be redistributed. So, the *characteristic function*  $v$  of our game corresponds to the cost savings corresponding to Equation (1).

Let  $x$  be an imputation.  $O^{(k)}$  receives  $x_k$  as payment. So  $O^{(k)}$  has incentive to be in the coalition if  $C_{\mathcal{SH}}^{(k)} - x_i \leq \text{cost}_{\text{local}}^{(k)}$ .

Moreover, for the rest of the document, we make two assumptions:

1. The algorithm building the scheduling assure the *monotonicity property*, i.e, the cost of scheduling jobs in  $\mathcal{J}$  using  $m$  machines is less than the cost of scheduling jobs in  $\mathcal{J}$  using  $m'$  machines if  $m < m'$ .

2. The total cost for scheduling among all organizations is the sum of the individual costs of each organization.

Note that the costs  $(\sum_J E_J)$  and  $(\sum_J C_J)$  respect these two constraints. Recall that, without the notion of organizations, they can be solved in polynomial time and respect the monotonicity property. Also note that the first assumption implies  $v(\mathcal{C}) \leq v(D)$  for every pair of subset  $\mathcal{C}, D \subseteq [N]$  such that  $\mathcal{C} \subseteq D$ .

## 4 Stable coalitions on the choreography enactment pricing game

This section is devoted to computing the imputation  $x$  for the coalition  $[N]$ .

Lets focus on organizations that will receive a non-null retribution. Consider the organizations where participating or not on a coalition does not alter the amount of savings.

**Lemma 4.** *Let  $[N]$  be a set of organizations and  $v$  be the characteristic function (corresponding to the cost savings). Let  $x$  be a feasible stable imputation. For each organization  $O^{(j)}$  in  $[N]$  such that  $v([N]) = v([N] \setminus \{j\})$ , we have  $x_j = 0$ .*

*Proof.* Since  $v$  respects the monotony constraints:  $v([N]) \geq v([N] \setminus \{j\})$  for any organization  $O^{(j)}$ .

First, we consider the case where all organizations  $O^{(j)}$  are such that  $v([N]) = v([N] \setminus \{j\})$ . It means that  $p(\{j\}) = \text{cost}^{([N])} - \text{cost}^{(-j)}$  for  $\forall j \in [N]$ . and the cost for  $[N]$  to execute jobs in  $\mathcal{J}^{(j)}$  is equals to the cost for organization  $O^{(j)}$  when it executes its own jobs alone. Since  $v([N]) = p([N]) - \text{cost}^{([N])}$ , we have  $v([N]) = 0$  and  $x_j = 0$  for  $\forall j \in [N]$ .

Second, we consider the case where at least an organization  $O^{(i)}$  is such that  $v([N]) > v([N] \setminus \{i\})$ . Let  $O^{(i^*)}$  be such an organization.

We prove this lemma by contradiction. Assume that there exists one organization  $j \in [N]$  such that  $v([N]) = v([N] \setminus \{j\})$  and  $x_j > 0$ .

We prove that  $x_{i^*} \leq v([N]) - v([N] \setminus \{i^*\})$ . If it is not the case, then organization  $O^{(k)} \in [N]$  could make an objection  $([N] \setminus \{i^*\}, y)$  against organization  $O^{(i^*)}$  such that  $y_\ell = x_\ell + \frac{x_{i^*} - (v([N]) - v([N] \setminus \{i^*\}))}{N-1}$  for  $\ell \in [N] \setminus \{i^*\}$ . Since  $\sum_{\ell \in [N] \setminus \{i^*\}} y_\ell = v([N] \setminus \{i^*\})$ , organization  $O^{(i^*)}$  cannot make a counter-objection against

$O^{(k)}$ . This means that  $x$  is not a feasible stable imputation, which leads to a contradiction.

Therefore,  $x_{i^*} \leq v([N]) - v([N] \setminus \{i^*\})$ . Organization  $O^{(i^*)}$  could make an objection  $([N] \setminus \{j\}, y)$  against organization  $O^{(j)}$  such that there exists an  $\varepsilon$  such that  $x_j > \varepsilon \geq 0$ ,  $y = x_{i^*} + \varepsilon$ , and  $y_k = x_k + \frac{x_j - \varepsilon}{N-2}$  for  $k \in [N] \setminus \{j, i^*\}$ . Note that for any  $k$ ,  $y_k > x_k$  since  $x_j > 0$ .

Now lets prove by contradiction that organization  $O^{(j)}$  cannot make a counter-objection  $(\mathcal{Q}, z)$ . Assume that organization  $O^{(j)}$  can make a counter-objection  $(\mathcal{Q}, z)$ . Let  $X$  be a set of organizations such that  $X = [N] \setminus \{j, i^*\}$ . By definition of counter-objection, we have  $\forall k \in X z_k \geq y_k$  and,  $\sum_{k \in X} z_k \geq v([N]) - (x_{i^*} + \varepsilon)$ .

Since  $v([N] \setminus \{i^*\}) = \sum_{k \in \mathcal{Q}} z_k$ , we obtain:

$$\begin{aligned} z_j &= v([N] \setminus \{i^*\}) - \sum_{k \in X} z_k \leq v([N] \setminus \{i^*\}) - v([N]) + (x_{i^*} + \varepsilon) \\ &\leq -x_{i^*} + (x_{i^*} + \varepsilon) \leq \varepsilon \end{aligned}$$

This contradicts the fact with  $0 \leq \varepsilon < x_j \leq z_j$ . Thus  $x$  is stable and this conclude the proof.  $\square \square$

Now, we focus on participating organizations that does change the amount of savings.

**Lemma 5.** Let  $[N]$  be a set of organizations and  $v$  be the characteristic function (corresponding to the cost savings). Let  $O^{(i)}$  and  $O^{(j)}$  be an organization such that  $v([N]) > v([N] \setminus \{i\})$  and  $v([N]) > v([N] \setminus \{j\})$ . Let  $\mathcal{O} = [N] \setminus \{j\}$  be a subset of organizations.

Let  $(\mathcal{O}, y)$  be an objection of  $O^{(i)}$  against  $O^{(j)}$ . In order to have a counter-objection to  $(\mathcal{Q}, z)$ , with  $\mathcal{Q} = [N] \setminus \{i\}$  of  $O^{(j)}$  against  $O^{(i)}$ , a sufficient condition is:

$$x_j - x_i \leq p(\{j\}) - p(\{i\}) - \text{cost}^{(-i)} + \text{cost}^{(-j)} \quad (2)$$

*Proof.* Assume that there is an objection  $(\mathcal{O}, y)$  of organization  $O^{(i)}$  against organization  $O^{(j)}$ . By definition of objection, we have that  $\forall k \in \mathcal{O}, y_k \geq x_k$  and  $y_i > x_i$ . Recall that  $v(\mathcal{O}) = p(\mathcal{O}) - \text{cost}^{(\mathcal{O})}$ .

Let  $X$  be a set of organizations such that  $X = [N] \setminus \{i, j\}$ .

Now, we will look for a counter-objection of organization  $O^{(j)}$  using  $(\mathcal{Q}, z)$ , where, for each  $k \in \mathcal{Q} \setminus \mathcal{O}$ ,  $z_k \geq x_k$  and for each  $k \in [N] \setminus \{i, j\}$ ,  $z_k \geq y_k$ . W.l.o.g, we can assume that  $\mathcal{Q} = X \cup \{j\}$  and  $k \in X$ ,  $z_k = y_k$ , otherwise, we can build another counter-objection  $\mathcal{Q}'$  such that  $k \in X$ ,  $z'_k = y_k$  and  $z'_j = z_j + \sum_{k \in X} (z'_k - z_k)$ .

By combining the definition of characteristic function  $v$ , we obtain:

$$v(\mathcal{Q}) - v(\mathcal{O}) = (p(\mathcal{Q}) - \text{cost}^{(\mathcal{Q})}) - (p(\mathcal{O}) - \text{cost}^{(\mathcal{O})})$$

By definition of function  $p$ , the previous equation can be rewritten

$$v(\mathcal{Q}) - v(\mathcal{O}) = p(\{j\}) - p(\{i\}) + \text{cost}^{(\mathcal{O})} - \text{cost}^{(\mathcal{Q})} \quad (3)$$

Since  $v(\mathcal{O}) = \sum_{k \in X} y_k + y_i$  and  $v(\mathcal{Q}) = \sum_{k \in X} z_k + z_j$ , Equation (3) can be rewritten as:

$$\sum_{k \in \mathcal{Q}} z_k - \sum_{k \in \mathcal{O}} y_k = p(\{j\}) - p(\{i\}) + \text{cost}^{(\mathcal{O})} - \text{cost}^{(\mathcal{Q})} \quad (4)$$

Since  $\sum_{k \in \mathcal{Q}} z_k - \sum_{k \in \mathcal{O}} y_k = z_j - y_i + \sum_{k \in X} (y_k - z_k)$ , it is sufficient that:

$$z_j - y_i \leq p(\{j\}) - p(\{i\}) - \text{cost}^{(\mathcal{Q})} + \text{cost}^{(\mathcal{O})} \quad (5)$$

We can notice that  $\text{cost}^{(\mathcal{O})} = \text{cost}^{(-j)}$  and  $\text{cost}^{(\mathcal{Q})} = \text{cost}^{(-i)}$ . From the definition of objection, it is sufficient to have:

$$x_j - x_i \leq p(\{j\}) - p(\{i\}) - \text{cost}^{(-i)} + \text{cost}^{(-j)}.$$

This concludes the proof of the lemma.  $\square$   $\square$

Lets focus on organizations that will receive non-null retribution, i.e., organizations on the coalition that does have impact on the amount of cost savings.

**Theorem 6.** Let  $[N]$  be a set of organizations and  $v$  be the characteristic function (corresponding to the cost savings). Let  $A$  be a subset of organizations  $\{j \in [N] : v([N]) > v([N] \setminus \{j\})\}$ . There exists a unique stable imputation  $x$  if  $x$  fulfills all the three following conditions:

1.  $\forall j \in [N] \setminus A, x_j = 0$
2.  $\forall j \in A, x_j = \text{cost}^{(-j)} + p(\{j\}) - \frac{1}{|A|} \cdot (\text{cost}^{(A)} + \sum_{k \in A} \text{cost}^{(-k)})$ ; and,
3.  $\forall j \in A, \text{cost}^{(-j)} + p(\{j\}) \geq \frac{1}{|A|} \cdot (\text{cost}^{(A)} + \sum_{k \in A} \text{cost}^{(-k)})$ .

*Proof.* Property (1) can be straightforward deduced from Lemma 4.

The bargaining set is the set of all imputations that do not admit a justified objection. So, if we apply Lemma 5 to  $i, j$  and then to  $j, i$ , we can derive that for any couple  $(i, j) \in [N]^2$ , we have

$$x_j - x_i = p(\{j\}) - p(\{i\}) - \text{cost}^{(-i)} + \text{cost}^{(-j)}.$$

Let  $O^{(j)}$  be an organization in  $A$ . Summing the previous equations, we obtain:

$$\sum_{k \in A} x_i - |A|x_j = \left( \sum_{k \in A} p(\{k\}) + \text{cost}^{(-k)} - |A|(\text{cost}^{(-j)} + p(\{j\})) \right) \quad (6)$$

From the properties of the value of the coalition and by computation, we can rewrite Equation (6) as:

$$\forall j \in A, x_j = \text{cost}^{(-j)} + p(\{j\}) - \frac{1}{|A|} \cdot \left( \text{cost}^{(A)} + \sum_{k \in A} \text{cost}^{(-k)} \right) \quad (7)$$

Property (3) can be deduced from the fact that  $x_j \geq 0$  and from Equation (7).  $\square$   $\square$

Now, we apply Theorem 6 to Example 1. Assume that organizations  $O^{(2)}$  and  $O^{(4)}$  form an alliance ( $[N] = \{2, 4\}$ ). Thus  $\text{cost}^{(-2)} = 1^\alpha$  and  $\text{cost}^{(-4)} = 7^\alpha$ . We obtain  $x_2 = x_4 = \frac{1}{2}(7^\alpha + 1^\alpha) - 4^\alpha$ . This means that in this alliance organization  $O^{(4)}$  pays  $p(\{4\})$ . From the alliance, it receive two payments. The first payment is due to the cost executing tasks assigned to it from other organizations in the alliance, and the second corresponds to the reward  $x_4$  for taking part in it. In any case, the imputation represents either a reduction in the costs of overloaded organizations (in this example, organization  $O^{(2)}$ ) or is a payment for the use of these resources by another organization (in this example, organization  $O^{(4)}$ ).

Now we will establish the relation between the price and the different costs when there is a stable imputation. Using Theorem 6, we will compute the lower bound for non-empty bargaining sets.

**Corollary 7.** *Let  $[N]$  be a set of organizations and  $v$  be the characteristic function  $v$  (corresponding to the cost savings). Let  $A$  be a subset of organizations  $\{j \in [N] : v([N]) > v([N] \setminus \{j\})\}$ . There exists a unique stable imputation  $x$  if  $p(A) \geq \text{cost}^{(A)}$ .*

*Proof.* From Theorem 6, we have:

$$\forall j \in A, \text{cost}^{(-j)} + p(\{j\}) \geq \frac{1}{|A|} \cdot (\text{cost}^{(A)} + \sum_{k \in A} \text{cost}^{(-k)}).$$

By summing these  $|A|$  inequations, we have:

$$\sum_{k \in A} (\text{cost}^{(-j)} + p(\{j\})) \geq \text{cost}^{(A)} + \sum_{k \in A} \text{cost}^{(-k)}.$$

The previous inequation can then be rewritten as:  $p(A) \geq \text{cost}^{(A)}$ .  $\square$   $\square$

These results constitutes the mathematical framework needed to devise an algorithm that, given a set of organizations  $[N]$ , determines whether an alliance is possible and, if possible, computes the imputation.

**Corollary 8.** *Let  $[N]$  be a set of organizations with their sets of jobs. If all organizations have as objective function  $(\sum_J C_J)$  or  $(\sum_J E_J)$ , then Algorithm 1 determines in polynomial time whether  $[N]$  can form a coalition and, if possible, returns the imputation vector.*

*Proof.* These objective functions respect the monotonicity property and an scheduling minimizing these objectives can be devise in polynomial time (see [2] for a polynomial algorithm for  $(\sum_J E_J)$ ). Therefore, we can apply Theorem 6 and Corollary 7, which are implemented by Algorithm 1.  $\square$   $\square$

Note that the same result straightforwardly applies to any scheduling problem whose objective function respects the two assumptions described in Section 3.2), as long as they can be solved in polynomial time.

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**Algorithm 1:** Coalition detection algorithm.

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**Input:** A set  $[N]$  of organizations, function  $v$  (corresponding to the cost savings), and  $\text{cost}^{(\cdot)}$ .

**Output:** (Whether there is a alliance or not and the imputation vector)

```
1 Compute the lowest cost schedule  $\mathcal{SH}$ ;  
2 forall the organizations  $O^{(o)} \in [N]$  do  
3   Compute the lowest cost schedule using only its own resources and its local cost  
    $(\text{cost}_{\text{local}}^{(o)} = p(\{o\}))$ ;  
4   Compute the lowest cost schedule using all resources except  $O^{(o)}$ 's resources and its cost  
    $(\text{cost}^{(-o)})$ ;  
5 Compute the lowest cost schedule using all the resources and its cost  $(\text{cost}^{([N])})$ ;  
6 forall the organizations  $O^{(o)} \in [N]$  do  
7   Compute  $p([N] \setminus \{o\})$   $(= \sum_{j \in [N], j \neq o} p(\{j\}))$ ;  
8   Compute  $v([N] \setminus \{o\})$   $(= p([N] \setminus \{o\}) - \text{cost}^{(-o)})$ ;  
9 Compute  $A = \{j \in [N] \mid v([N]) > v([N] \setminus \{j\})\}$ ,  $p(A)$ ,  $\text{cost}^{(A)}$  and  $\sum_{k \in A} \text{cost}^{(-k)}$ ;  
10 if  $p(A) < \text{cost}^{(A)}$  then  
11   return  $(\text{coalition}=\text{false}, \text{imputation}=\emptyset)$   
12 forall the organization  $O^{(o)} \in A$  do  
13   compute  $x_o$  according to Equation (2) of Theorem 6;  
14 return  $(\text{coalition}=\text{true}, \text{imputation}=x)$ 
```

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## 5 Related work

The problem of scheduling jobs on independent, selfish organizations sharing a common infrastructure — known as the Multi Organization Scheduling Problem or MOSP— was first studied by Pascual et al. [12,20] and was then extended to include notions from game-theory by Cohen et al. [7,8]. The original studies does not include the ability to form coalitions; MOSP only allows rebalancing the jobs between organizations, as long as no organization presents a performance degradation according to their its own performance objective.

The choreography enactment pricing game is similar to fair resource allocation and networking games. Fragnelli et al. [13] studied a related cooperative game that they called the shortest path games. Their game models agents willing to transport a good through a network from a source to a destination. Using a graph model, and letting agents to control the nodes, they have studied how profits should be allocated according to the core of the cooperation.

Maintaining the assumption of  $s$ -veto players, Voorneveld and Grahn [22] extended the shortest path game and proved that the core allocations coincide with the payoff vectors in the strong Nash equilibria of the associated non-cooperative shortest path game.

Several subsequent papers [5,18] studied computability and complexity aspects of this game. Some properties of graphs and games guaranteeing the existence of a core have been proposed and the computability complexity of computing cores have been established (NP-complete and #P-complete). Other variants (different payoffs and players controlling arcs) have been considered, but mostly focusing on the existence and complexity of cores, whereas this work mostly focus on the construction of the bargaining set in polynomial time.

The flow game can be view as maximum multicommodity flow problem in a cooperative setting. This model can be used to identify the set of demands to satisfy and to route this demand on the network. In this context, players own network resources and share a capacity to deliver commodities. Kalai et



al. [14] first considered flow games for network with a single commodity, where a unique player owns an arc. Several studies (for example, [1, 11, 16]) extended this seminal work. Those extensions encompass variations on the number of arcs a player can control, if the player controls all or a part of the capacity of the arc, if players control vertices, etc. Those papers mainly focus on how to obtain the optimal flow in the network and then on how to allocate the revenue using core allocation techniques (since those games have non-empty cores).

## 6 Conclusion

This work presents a game-theoretic model for the problem we call the choreography enactment pricing problem. Service providers must enact, i.e., assign all jobs that composes the users' applications (service compositions) to resources of its own or subcontract resources from other service providers. Each organization must pay a price to execute the jobs from its users. This price depends on a given cost function that is common to all organizations (e.g., energy costs). In order to reduce its costs, a set of organizations can form alliances (coalitions) to share its resources and reduce its costs.

From an economic point of view, the lack of regulations may create alliances that can control the prices and/or limit competition. Our study of the choreography enactment pricing problem as a cooperative game characterized the mathematical conditions needed for the creation of stable coalitions. From this characterization we devised an algorithm to detect if such alliances can be formed.

The study of the bargaining set of this problem suggests that this problem may be related to the notion of *truthful mechanism* from algorithmic mechanism design in game theory. To the best of our knowledge, there is no study about this relation and it would be an interesting future work.

Also as future work, we will consider this problem with a broader set of cost functions (such as the total makespan). For now, our results cannot be apply cost functions that does not respect monotony constraints. One of the first step is to consider the load as the cost (i.e. the sum of all jobs executed in the same organization). Recently, Azar [4] designed a polynomial randomized 2-approximation algorithm for minimizing makespan using restricted-related machines. They use technique to find an optimal fractional solution [15] and they modify optimal fractional solution that the load assigned to machine is monotone. Using this result and from the expectation linearity, all our results can be applied. It means that to find an imputation  $x$  such that, for all  $k$  ( $1 \leq k \leq N$ ),  $\mathbb{E}[C_{\mathcal{SH}}^{(k)}] - x_i \leq \mathbb{E}[\text{cost}_{\text{local}}^{(k)}]$  if such vector exists can be computed in polynomial time. This technique can may be help to solve for the formation of coalition considering makespan as the cost.

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